8. A woodman fells a tree 2 feet in diameter, cutting half way through from each side. The lower face of each cut is horizontal, and the upper face makes an angle of 45° with the horizontal. How much wood does he cut out?

[Selected from *Byerly's Integral Calculus.*]

I. Solution by CHARLES E. MYERS, Canton, Ohio.

Conceive each part removed to be generated by the motion of a right-angled triangle, moving so that its base is in a cross section of the tree and perpendicular to the axis of the tree, the sides of the triangle varying and the angles remaining constant$=45°$. Let $z=$the altitude of the triangle at any time; $V=2v=$entire volume removed, and put $2r=2$ feet.

Putting the origin of co-ordinates at the circumference, we have,

$v=\int_0^{2r}\frac{1}{2}yzdx$. From the circle, $y\ (2r-x)^{\frac{1}{2}}$, and since the angle$=45°$, $z=y$ at all times. Substituting these values of y and z in the above equation and integrating, we have,

$v=\int_0^{2r}\frac{1}{2}(2rx-x^2)dx=\frac{2}{3}r^3$, and doubled, gives $V=\frac{4}{3}r^3=1\frac{1}{3}$ cu. ft.

II. Solution by J. R. BALDWIN, A. M., Professor of Mathematics in the Davenport Business College, Davenport, Iowa.

Let $r=1$ foot$=$the radius of the tree, AB the common edge of the two cuts, $\theta=$the angle which the radius makes with AB, $\varphi=45°$. For the element of volume, we have, $2r\cos\theta.r\sin\theta\tan\varphi.d(r\sin\theta)$.

$$\therefore \text{ Volume } =4r^3\tan\varphi\int_0^{\frac{1}{2}\pi}\cos^2\theta\sin\theta d\theta=-4r^3\left[\frac{\cos^3\theta}{3}\right]_0^{\frac{1}{2}\pi}=\frac{4}{3}r^3=1\tfrac{1}{3}\text{cu.ft.}$$

Also solved by *M. C. Stevens*, *P. H. Philbrick*, *Seth Pratt*, *Alfred Hume*, *H. W. Draughon*, *H. C. Whitaker*, *G. B. M. Zerr*, *W. L. Harvey*. and *P. S. Berg*.

PROBLEMS.

16. Proposed by F. P. MATZ, M. Sc.,Ph.D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with regard to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

17. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

To find the volume generated by revolving a circular segment whose base is a given chord, about any diameter as an axis.

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